# Is the Extension of Trading Hours Always Beneficial? An Artificial Agent-Based Analysis

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**Abstract:** The extension of trading hours to provide more trading opportunities and improve price efficiency has increasingly been discussed. However, currently, there is limited trading activity during the stock market's extended-hours trading session. Thus, we should examine whether the extension of trading hours is still effective for creating more trading opportunity and price efficiency even if there are few market participants during the extended session. For this study, we build an agent-based market model based on that of Brock and Hommes (1998) and analyze the effect of extending trading hours. We find that although extending trading hours could increase daily trading volume, it could distort price formation and trade opportunity if market participants are limited during the extended-hours session. Specifically, the extension could result in more concentrated trading in the opening session, wider divergence between market prices and the fundamental value of stocks, and higher return volatility (especially at the open).

#### 1. Introduction

Recently, the extension of trading hours for stocks has increasingly been discussed. In several markets (e.g., NYSE and NASDAQ), both pre-market and after-hours trading sessions have already been introduced, and the Tokyo Stock Exchange is considering extending trading hours by introducing extended-hours sessions and/or shortening the midday recess. The extension of trading hours is intended to provide more trading opportunities and improve price efficiency (Osaki, 2014).

Periodical market closures can cause significant negative effects on price efficiency and trading opportunity. First, periodical market closures might impede stock prices from incorporating public and private information. Kyle (1985), Glosten and Milgrom (1985), Foster and Viswanathan (1990), and Easlay and O'Hara (1992) show public and private information accumulates overnight while information asymmetry declines over the course of trading periods. These studies suggest market closures may induce a delay in the incorporation of information into stock prices, which can widen divergence between stock prices and their fundamental values. Second, periodical market closure may cause excessive price fluctuations, especially at the beginning and end of the trading session on an intraday basis. Wood et al. (1985) and Harris (1986) find that a

Therefore, the extension of trading hours is likely to mitigate market inefficiencies caused by market closures, i.e., extending trading hours could lower divergence between market prices and fundamental values, lower stock return volatility (especially at the open and close), increase daily trading volume, and ease concentration of trading activity at the open and close.

However, limited investor participation during extended hours is a concern regarding the effect of extending trading hours. In the U.S., where extended trading hours have already been introduced, the trading volume per unit of time during extended hours is less than 5% as that during regular trading hours (Barclay and Hendershott, 2004). Although trading during extended hours allows investors to quickly react to after-market news, market prices are less efficient during extended hours compared to regular hours due to reduced liquidity (Barclay and Hendershott, 2003). It is quite uncertain

standard deviation of returns is especially high at the open and close of trading; this U-shaped pattern of return volatility is also found in non-U.S. markets (Hamao and Hasbrouck, 1995; Abhyankar et al., 1997). Third, periodical market closures can cause skewed trading activity, i.e., trading is concentrated at the beginning and end of trading sessions. Jain and Joh (1988) document a U-shaped intraday pattern of trading volume; trading volume is especially high at the open and close of trading. Finally, obviously, periodical market closures could reduce investors' trading opportunities.

<sup>&</sup>lt;sup>1</sup> This paper is an edited summary of a longer paper available at the SSRN (http://ssrn.com/abstract=2543412)

whether extending trading hours would mitigate the market inefficiencies if there are few market participants during extended-hours sessions.

In this study, we uncover the effect of extending trading hours assuming limited market participation during the extended-hours sessions.

There are a few studies that empirically analyze the effect. For example, Houstion and Ryngaert (1992) find reductions in NYSE trading hours had little effect on return volatility and trading volume during the week the reductions occurred, but did have an effect on the distribution of return volatility and trading volume during the week. The study of Fan and Lai (2006) reveals significant change in the intraday pattern of return volatility and trading volume could not be observed after extending the trading session of the Taiwan Stock Exchange by 1.5 hours. Although these studies might indicate the market inefficiencies are not easily mitigated by a change in trading hours, the result might also be due to an insufficient change in trading hours.

The drawback of empirical analyses on the extension of trading hours is there is no perfect sample with which to compare prices and trading behavior between a market with extended-hour sessions and one without extended hours. There is an obvious limitation with respect to showing the effect of extended hours via empirical analyses.

On the other hand, model-based analysis allows for direct comparison of price behavior and trading activity between a market with extended hours and one without extended hours. In addition, we can easily understand underlying reasons the extension of trading hours is effective or ineffective when there are limited market participants during extended-hours sessions.

Thus, in this study, we perform model-based analysis to analyze the effect. We specifically designed our simulation model based on that of Brock and Hommes (1998) rather than existing models that are built for analyzing the effect of market closures; a few studies built analytical models to analyze the effect of market closure on price and trading volume as well. Brock and Kleidon (1992) expand on Merton's model to analyze the effect of market closures on the trading concentration at the open and close. To analyze the intraday pattern of price and trading behavior, Hong and Wang (2000) develop a competitive market model with periodic closures where investors trade for allocation and informational reasons. The model by Brock and Kleidon (1992) is in a partial equilibrium setting and cannot show

the equilibrium dynamics between returns and trading volume. More importantly, since both models are analytically tractable general equilibrium models, in these models it is assumed there are enough market participants; in other words, the models cannot analyze the effect of the number of market participants on market prices and trading activity. Therefore, these models do not allow for analyzing the effect of extended-hours sessions with limited market participants. On the other hand, the model by Brock and Hommes (1998) is a simple simulation-based model with evolutional dynamics; the model's strengths are simplicity and high flexibility (high scalability). Thus, in this study, we expand the model by incorporating extended trading hours with limited investor participation.

First, we show the extended model can reproduce the U-shaped intraday pattern of return volatility and volume, gradual incorporation of fundamental information during regular trading hours, and two important stylized facts: fat-tailed returns and clustered volatility. Next, we examine the effect of illiquid extended-hours sessions. Specifically, we compare the following two cases: Case 1 includes investors who can only trade during regular hours and case 2 involves investors who can trade 24 hours per day, but there are limited market participants during extended-hour sessions. Then, we examine differences in the following three factors between the cases: the deviation between stock prices and fundamental values, volatility of stock returns (especially at the open and close), and trading volume (especially at the open and close). If illiquid extended-hours sessions mitigate the negative effect induced by market closure, the deviation between market price and fundamental value, return volatility, and trading concentration at the open and close should be smaller, and daily trading volume should be higher for case 2 than for case 1.

#### 2. Market Model

In this section, we explain the agent-based market model used. We built a simple artificial market model based on that of Brock and Hommes (1998), including the extended-hours session with limited investor participants.

#### 2.1 Basic Model

#### **Price Determination Process**

Agents can either invest in a risk free asset or in a risky asset. The risk free asset has perfect supply elasticity, and in the short-term investment horizon, the interest rate and dividend yield are irrelevant; therefore,

we suppose the interest rate is zero and the risky asset (e.g. a stock) pays no dividend. Let  $P_t$  be the price per share of the risky asset at time t.

First, we show a price determinant process when there is no periodic market closure. The process is similar to that of Brock and Hommes (1998) under the condition the assets pay no dividend and the interest rate is zero. Agents are myopic mean-variance maximizers, so the demand  $z_{i,t}$  per trader i for the risky asset is calculated:

$$z_{i,t} = \frac{E_{i,t}[\frac{\mathbf{p}_{t+1}}{p_t} - 1]}{aV_{i,t}[\frac{\mathbf{p}_{t+1}}{p_t} - 1]}$$

 $E_{i,t}$  and  $V_{i,t}$  denote the "beliefs" (forecasts) of trader i about conditional expectation and conditional variance of return  $p_{t+1}/p_{t}$ -1, and a is the risk aversion parameter. Bold face variables denote random variables at date t+1. The conditional variance  $V_{i,t} = \sigma^2$  is assumed to be equal and constant for all investors.

$$z_{i,t} = \frac{E_{i,t}[\frac{p_{t+1}}{p_t} - 1]}{a\sigma^2}$$
 (1)

Let  $z^s$  denote the supply of outside risky shares per investor, and it is assumed to be constant. When there are N traders, equilibrium of demand and supply yields:

$$\sum_{i} \frac{E_{i,t} \left[ \frac{\mathbf{p}_{t+1}}{p_t} \right] - 1}{a \sigma^2} = Z^{s}$$

Brock and Hommes (1998) focus on the special case of zero supply of outside shares, i.e.  $z^s = 0$ , for which the Walrasian market clearing price satisfies:

$$p_t = \frac{1}{N} \sum_i E_{i,t}[\mathbf{p}_{t+1}] \tag{2}$$

The volume of trade TV at time t is given by:

$$TV_{t} = \frac{1}{2} \sum_{i} \left| z_{i,t} - z_{i,t-1} \right| \tag{3}$$

# Heterogeneous Beliefs and Evolutionary Selection of Strategies

Regarding traders' heterogeneous expectations about future prices, we basically follow the heterogeneous expectation process of Brock and Hommes (1998). All traders are assumed to be able to derive the fundamental price  $p_t^*$  that would prevail in a perfectly rational world. The fundamental price continuously reflects the upcoming fundamental news; the value is assumed to be varied over time as:

$$p_{t+1}^{*} = p_{t}^{*} e^{W}$$
 $W \sim N(0, \sigma_{f})$  (4)

Traders believe that in a heterogeneous world prices

may deviate from their fundamental value  $p_t^*$ . It is convenient to introduce the deviation from the fundamental price:

$$x_{\iota} = p_{\iota} - p_{\iota}^{*}$$

Following the "four investor type" model of Brock and Hommes (1998), we assume the model has four investor types: fundamentalists (denoted as  $X_1$ ), trend followers (denoted as  $X_2$ ) who allow price deviation from fundamental value, and two investor types with purely biased beliefs: optimists (denoted as  $X_3$ ) and pessimists (denoted as  $X_4$ ), who expect a constant price above (optimists) or below (pessimists) the fundamental price.

$$\begin{split} E_i[p_{t+1}] &= p_{t+1}^* \qquad i \in X_1 \text{ Fundamentalists} \\ E[p_{t+1}] &= p_{t+1}^* + g \cdot x_t \qquad i \in X_2 \text{ Trend followers} \\ E[p_{t+1}] &= p_{t+1}^* \cdot (1+d) \quad i \in X_3 \text{ Optimists} \\ E[p_{t+1}] &= p_{t+1}^* \cdot (1-d) \quad i \in X_4 \text{ Pessimists} \\ \text{Where } d > 0 \end{split}$$

The evolutionary part of the model, describing how beliefs are updated over time, follows the endogenous selection of forecasting rules introduced by Brock and Hommes (1997); the probability  $\Pr_i\{i \in X_i\}$  is given by:

$$\Pr_{t}\{i \in X_{s}\} = \begin{cases} P_{upd} \cdot \frac{\exp(\beta U_{s,t-1})}{\sum_{s} \exp(\beta U_{s,t-1})} + (1 - P_{upd}) & \text{if } i \in X_{s} \text{ at time } t - 1\\ P_{upd} \cdot \frac{\exp(\beta U_{s,t-1})}{\sum_{s} \exp(\beta U_{s,t-1})} & \text{if } i \notin X_{s} \text{ at time } t - 1 \end{cases}$$

Where  $0 \le P_{upd} \le 1$ 

 $U_{s,t}$  is the fitness measure of strategy s evaluated at time t. A natural candidate for evolutionary fitness is realized profits, given by:

$$U_{s,t} \equiv (p_t / p_{t-1} - 1)(\frac{E[p_t] / p_{t-1} - 1}{a\sigma^2})$$

#### 2.2 Extension of the Model

# Price Determination Process during the Extended Hours Session

Now, we consider the case when there is a periodic market closure or the extended-hours session has imperfect market liquidity.

We assume investors can trade a risky asset 24-hours day. A daily trading session consists of a regular-hours session (during which there is successive  $N_{regular}$  time steps) and an extended-hours session (successive  $N_{extended}$  time steps). We call the first time step of the regular-hours session and the last time step of the regular-hours session the "opening session" and the

"closing session," respectively. In addition, we call a step subsequent to the opening session the "subsequent session." All the investors trade the asset during the regular-hours session; on the other hand, only a limited number of investors trade outside the regular hour session (during the extended-hours session).  $X_{both}$  denotes investors who trade the asset during both the regular-hours and extended-hours sessions,  $X_{regular}$  denotes investors who trade the asset only during the regular-hours session, and  $P_e$  denotes the ratio of investors who trade the asset during the extended-hours session ( $P_e = 0$  means all the investors trade only during the regular-hours session).

During the regular-hours session, except for the last time step of the regular-hours session, the demand  $z_{i,r}$  can be given by (1). Thus, a price of the risky asset is given by (2). On the other hand, the price is determined differently in the closing session and during the extended-hours session.

The demand  $z_{i,t}$  of investors who trade only during the regular-hours session is set to be constant during the extended-hours session; this setting means there is no trading activity among these investors during the extended-hours session. Thus, the demand  $z_{i,t}$  during the extended-hours session is given by:

$$z_{i,t} = \begin{cases} \frac{E_{i,t}[p_{t+1}/p_t] - 1}{a\sigma^2} & i \in X_{both} \\ z_{i,t-1} & i \in X_{regular} \end{cases}$$
 (5)

Thus, equilibrium of demand and supply yields:

$$\sum_{i \in X_{both}} \frac{E_{i,t}[p_{t+1}/p_t] - 1}{a\sigma^2} + \sum_{i \in X_{regular}} z_{i,t-1} = 0$$

Thus, market clearing price satisfies:

$$p_{t} = \frac{\sum_{i \in X_{both}} E_{i,t}[p_{t+1}]}{N_{both} - a\sigma^{2} \sum_{i \in X_{regular}} Z_{i,t-1}}$$
(6)

# Price Determination Process in the Closing Session

As discussed in the arguments of Brock and Kleidon (1992) and Hong and Wang (2000), investors who trade only during the regular-hours session assume overnight risk in the closing session. The risk aversion parameter for these investors could be higher than that for investors who also trade during the extended-hours session. To incorporate this possibility, we define the risk aversion parameter  $b_i$  ( $b_i$ >=a) at the close by:

$$b_{i} = \begin{cases} aB_{i} i \in X_{regular} \\ a \quad i \in X_{both} \end{cases}$$
 (7)

Where  $B_{i}$ -1 is assumed to follow exponential distribution:

$$B_i - 1 \sim Exp(1/\lambda) \tag{8}$$

Therefore, the demand  $z_{i,t}$  is given by:

$$z_{i,t} = \frac{E_{i,t}[p_{t+1}/p_t] - 1}{b_i \sigma^2}$$
(9)

Thus, equilibrium of demand and supply yields:

$$\sum_{i} \frac{E_{i,t}[p_{t+1}/p_{t}]-1}{b_{i}\sigma^{2}} = 0$$

The market clearing price satisfies:

$$p_{t} = \sum_{i} \frac{E_{i,t}[p_{t+1}]}{b_{i}} / \sum_{i} \frac{1}{b_{i}}$$

# 3. Simulation Results

#### 3.1 Simulation Settings

We define 24 time steps as comprising one trading day (1 time step per hour). We denote each time step as T1, T2...T24. The first 6 time steps each day comprise the regular-hours session and the remaining 18 time steps comprise the extended-hours session. We set the simulation length at 250 days, the number of investors at 100, the risk aversion parameter in Formula 1 (a) at 1, the volatility parameter for the fundamental value  $(\sigma_f)$  at 0.01, the strategy update parameter ( $\beta$ ) at 1, and the constant estimated return volatility in Formula 1 ( $\sigma^2$ ) at 1. We choose to let  $P_{e}$  (the ratio of investors who trade during the extended-hours sessions) =  $\{0, 0.1, 0.2...0.9,$ 1.0}, d (the parameter of biased estimation) =  $\{0, 0.1, 0.2,$ 0.3, 0.4, g (the parameter of trend chasing) =  $\{0, 0.25,$ 0.5, 0.75, 1.0, 1.25, 1.5},  $P_{upd}$  (the parameter of strategy update speed) =  $\{0, 0.1, 0.2...0.9, 1.0\}$ , and  $\lambda$  (the parameter of overnight risk) =  $\{1, 3, 5, 7, 9\}$ . All statistics and the following figures use an average of 100 simulation runs.

#### 3.2 Adequate Parameters

In this section, we examine whether our model can reproduce a U-shaped intraday pattern of return volatility and trading volume when there is periodic market closure. In addition, we examine whether our model explains two important stylized facts: fat-tailed returns and clustered volatility, which are reported by several prior studies (e.g. Mandelbrot, 1963, 1997; Pagan, 1996; Cont et al., 1997). Several artificial market model studies (e.g. LeBaron, 2006; Chen et al., 2012) examine whether their model can explain these stylized facts to verify their artificial market models. The simulation analyses in Section 3.2 are performed under the condition there is periodic market closure.

As a result<sup>2</sup>, we found the model replicates the U-shaped intraday pattern of trading volume and return volatility, gradual incorporation of fundamental information during regular-hours sessions, and the two statistically existing stylized facts, under the conditions:

- (1) There are strong trend followers (at least, g > 0.5)
- (2) There are biased traders (d>0)
- (3) Investors update their strategy, but not excessively (at least  $0 < P_{upd} < 0.5$ ).

# 3.3 The Effect of Extending Trading Hours

The parameters that satisfy the conditions mentioned in Section 3.2 can be regarded as adequate model parameters verified by the stylized facts, intraday volume and volatility patterns, and the gradual incorporation of fundamental information. Specifically, we show the result when we set d, g,  $\lambda$ , and  $P_{upd}$  at 0.2, 1.25, 5, and 0.2, respectively. However, the implication of the simulation result is invariant regardless of the parameter settings as long as the settings satisfied the above-mentioned conditions.

We change  $P_e$  (the ratio of market participants who trade during the extended-hours session) from 0 to 1 at an interval of 0.1. Then, we examine the effect of an extension of trading hours on 1) price efficiency, which is evaluated by the deviation between stock prices and fundamental values, 2) return volatility during the regular-hours session and the U-shaped intraday pattern of return volatility, and 3) daily trading volume and trading concentration at the open and close.

# 3.3.1 Price Efficiency

We examine whether the deviation between prices and fundamental values is lowered by extending trading hours, even if there are limited market participants during the extended-hours session. We change  $P_e$  from 0 to 1 and calculate a difference between market price and fundamental value, which is defined by  $|\log(P_t/P_t^*)|$ , during the regular-hours sessions and that during the extended-hours sessions. Figure 1 show the simulation results.

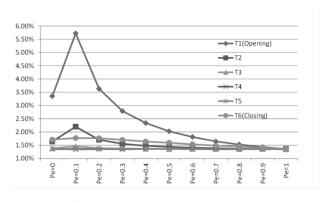
The result, shown in Figure 1(a), reveals when there are enough market participants during the extended-hours session (when  $P_e \geq 0.4$ ), the stock price is closer to the fundamental value than when there is periodic market closure. However, when there is not enough market participation during the session (especially when  $P_e \leq 0.2$ ), the stock price diverges

<sup>2</sup> The detail of the results analysis is presented in the full paper available at the SSRN (<a href="http://ssrn.com/abstract=2543412">http://ssrn.com/abstract=2543412</a>).

from the fundamental value more than when there is periodic market closure. These results suggest the extension of trading hours could result in lower price efficiency when there are not enough market participants during the extended-hours session. Interestingly, not only during the extended-hours session but also during regular hours, price diverges more from fundamental value if  $P_e$  $\leq$  0.2 than when there is a periodic market closure. The result reveals stock price diverges more from fundamental value during the last time step of the extended session, indicating illiquid trading has been disturbing an incorporation of fundamental information into the asset price during the extended session. The divergence is narrowed after the opening bell, indicating an increase in market participants improves price efficiency. However, as shown in Figure 1(b), the widened divergence is not completely corrected at the open; the negative effect of the illiquid extended-hours session on price efficiency remains even after the opening bell.



(a) Regular-Hours and Extended-Hours Sessions



(b) Each Time Step during the Regular-Hours Sessions Notes: "Regular-Hours Session" and "Extended-Hours Session" in Figure 4(a) represent the time-series average of the deviation during the regular-hours and extended-hours sessions, respectively. "T1," "T2"..."T6" in Figure 4(b) represent an

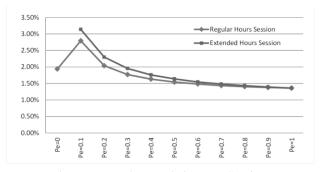
average value of the deviation in periods T1, T2...T6, respectively.

Figure 1 The Deviation from Fundamental Value

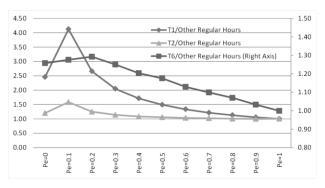
#### 3.3.2 Return Volatility

We examine whether return volatility during the regular-hours session (especially at the open and close) is decreased by the extension of trading hours, even if investors rarely trade during the extended-hours sessions. We calculate a ratio of return volatility at the open to that during the other regular-hours session, and a ratio of return volatility at the close to that during the other regular-hours session. We examine whether these ratios and the return volatility during the regular-hours session decreases as  $P_e$  increases. If so, we can say the extension of trading hours decreases return volatility and weakens the U-shaped intraday pattern of return volatility even if there are few market participants during the extended-hours sessions.

The result, shown in Figure 2, reveals when there are enough market participants during the extended-hours session (when  $P_e \ge 0.3$ ), return volatility during the regular-hours session is lower, and return volatility at the open and close is closer to that during the other regular-hours sessions versus when there are periodical market closures. However, when there are not enough market participants during the extended-hours session (when  $P_e < 0.3$ ), return volatility during the regular-hours session and the ratio of return volatility at the open to that during the other regular-hours session are higher than when there are periodic market closures. As discussed in Section 3.3.1, if market participants are limited during the extended-hours session, price diverges more from the fundamental value during the session. Since this divergence from fundamental values is corrected in the opening and subsequent sessions, return volatility increases in the opening and subsequent sessions.



(a) Regular-Hours and Extended-Hours Sessions



#### (b) Intraday Pattern of Return Volatility

Notes: Graphs "Regular-Hours Session" and "Extended-Hours Session" in Figure 2(a) represent average volatility during the regular-hours and extended-hours sessions, respectively. Graphs "T1/Other Regular Hours," "T2/Other Regular Hours," and "T6/Other Regular Hours" in Figure 2(b) represent the ratios of average volatility in the opening session (T1) to that during the other regular-hours session, average volatility in the subsequent session (T2) to that during the other regular-hours session, and average volatility in the closing session (T6) to that during the other regular-hours session, respectively.

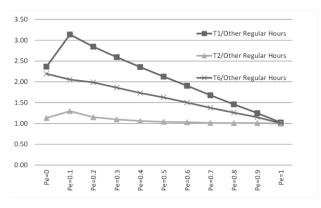
Figure 2 Return Volatility

#### 3.3.3 Trading Volume

We examine whether illiquid extended-hours sessions increase daily trading volume and weaken trading concentration at the open and close (the U-shaped intraday pattern of trading volume). We calculate daily trading volume, a ratio of average volume at the open to that during the other regular-hours session, and a ratio of average volume at the close to that during the other regular-hours session. The result, shown in Figure 3(a), reveals the volume per day increases as  $P_e$  increases, indicating the extension of trading hours increases daily trading volume even if there are limited market participants during the extended-hours session. However, as shown in Figure 3(b), the ratio of average volume at the open to that during the other regular-hours session is higher than when there are periodical market closures if there are not enough market participants during the extended-hours session ( $P_e$ <0.5). This result indicates trading concentration at the open is increased by the extension of trading hours if there are few market participants during the session. The increased concentration at the open could be attributed to wide divergence between market prices and their fundamental values just before the opening bell. The large gap between market prices and fundamental values just before the opening bell results in increased trading activity at the open, which results in a much smaller gap. Therefore, the trading concentration at the open is not lowered by the extension of trading hours when there are not enough market participants during the extended-hours session.



### (a) Daily Trading Volume



### (b) Trading Concentration

Notes: Figure 3(a) shows average volume per day. Graphs "T1/Other Regular Hours," "T2/Other Regular Hours," and "T6/Other Regular Hours" in Figure 3(b) represent the ratios of average volume in the opening session (T1) to that during the other regular-hours session, average volume in the subsequent session (T2) to that during the other regular-hours session, and average volume in the closing session (T6) to that during the other regular-hours session, respectively.

**Figure 3 Trading Volume** 

# 4. Conclusion

We built a simple agent-based market model based on that of Brock and Hommes (1998), which includes the extended-hours session with limited investor participants. By utilizing the model, we examine whether the extension of trading hours is effective for creating more trading opportunity and price efficiency, even if there are few market participants during the extended-hours session.

We find extending trading hours increases daily trading volume. However, it could increase a divergence between market prices and fundamental values not only during the extended-hours session, but also during the regular-hours session. In addition, we find the extension could result in higher return volatility during the regular-hours session and does not mitigate the U-shaped intraday pattern of return volatility when there are not enough market participants during the extended-hours session. Finally, the extension could increase trading concentration at the open.

The simulation reveals the negative impact of the illiquid extended-hours session is observed if there are limited market participants during the extended-hours session (if there is less than 30% as much market participation during the extended-hours session as during the regular-hours session), and there are biased investors and strong trend followers. It seems the extension rarely causes the aforementioned negative impact on trading activity and price formation in actual stock markets.

However, in terms of illiquidity during the extended-hours session, market participation during the extended-hours session is actually quite limited: there is less than 5% as much trading per time unit in after-hours sessions versus regular trading sessions in the U.S. stock market (Barclay and Hendershott, 2004). In terms of trend followers and biased traders, our analysis (and previous study) shows the stylized facts and the U-shaped intraday pattern of volume and volatility can be observed as long as there are strong trend followers and biased traders. Thus, it is highly possible the extension of trading hours has a negative impact on trading activity and price formation, in actual stock markets.

In sum, our findings give important indication about increasing discussion regarding the extension of trading hours. As argued in previous studies, periodic market closures could distort trading activity and price formation, e.g. high return volatility and trade concentration especially at the open and close, and delay incorporation of fundamental information into prices. It seems this problem can easily be solved by extending trading hours. However, our results suggest the problem is not so simple; the extension of trading hours could disturb price formation and trading activity if market participation during the extended-hours session is limited. Since market participation during the extended session is

limited in actual stock markets, this finding raises the possibility that the extension of trading hours has a negative impact on actual stock markets. Our finding emphasizes the importance of debating whether many investors will trade during the extended-hours session, and how to encourage investors to trade during the session before extending trading hours.

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